

General Certificate of Education
January 2005
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 2

MPC2

Friday 21 January 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

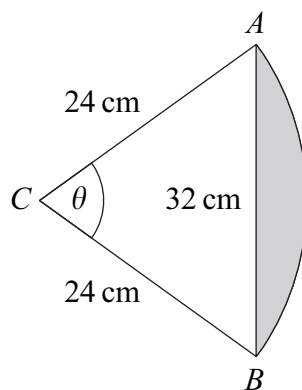
1 A curve is defined for $x > 0$ by the equation $y = x + \frac{2}{x}$.

(a) (i) Find $\frac{dy}{dx}$. (3 marks)

(ii) Hence show that the gradient of the curve at the point P where $x = 2$ is $\frac{1}{2}$. (1 mark)

(b) Find an equation of the normal to the curve at this point P . (4 marks)

2 The diagram shows a triangle ABC and the arc AB of a circle whose centre is C and whose radius is 24 cm.



The length of the side AB of the triangle is 32 cm. The size of the angle ACB is θ radians.

(a) Show that $\theta = 1.46$ correct to three significant figures. (3 marks)

(b) Calculate the length of the arc AB to the nearest cm. (2 marks)

(c) (i) Calculate the area of the sector ABC to the nearest cm^2 . (2 marks)

(ii) Hence calculate the area of the shaded segment to the nearest cm^2 . (3 marks)

3 An arithmetic series has fifth term 46 and twentieth term 181.

- (a) (i) Show that the common difference is 9. (3 marks)
- (ii) Find the first term. (1 mark)
- (b) Find the sum of the first 20 terms of the series. (2 marks)
- (c) The n th term of the series is u_n .

Given that the sum of the first 50 terms of the series is 11525, find the value of

$$\sum_{n=21}^{50} u_n \quad (2 \text{ marks})$$

- 4 (a) Write \sqrt{x} in the form x^k , where k is a fraction. (1 mark)
- (b) Hence express $\sqrt{x}(x-1)$ in the form $x^p - x^q$. (2 marks)
- (c) Find $\int \sqrt{x}(x-1) dx$. (3 marks)
- (d) Hence show that $\int_1^2 \sqrt{x}(x-1) dx = \frac{4}{15}(\sqrt{2} + 1)$. (3 marks)

5 (a) Given that

$$\log_a x = 3 \log_a 6 - \log_a 8$$

where a is a positive constant, show that $x = 27$. (3 marks)

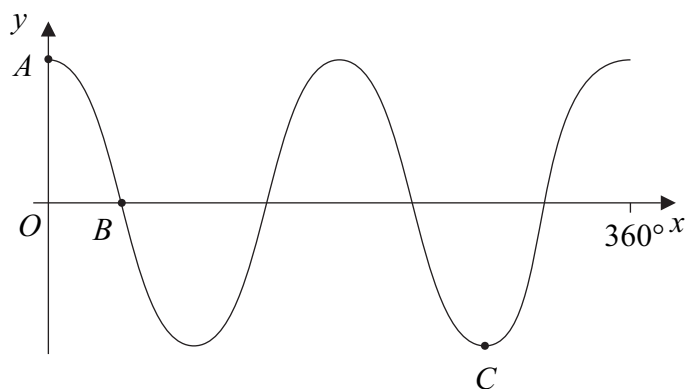
- (b) Write down the value of:
- (i) $\log_4 1$; (1 mark)
- (ii) $\log_4 4$; (1 mark)
- (iii) $\log_4 2$; (1 mark)
- (iv) $\log_4 8$. (1 mark)

- 6 (a) (i) Using the binomial expansion, or otherwise, express $(2 + x)^3$ in the form $8 + ax + bx^2 + x^3$, where a and b are integers. (3 marks)
- (ii) Write down the expansion of $(2 - x)^3$. (2 marks)
- (b) Hence show that $(2 + x)^3 - (2 - x)^3 = 24x + 2x^3$. (2 marks)
- (c) Hence show that the curve with equation

$$y = (2 + x)^3 - (2 - x)^3$$

has no stationary points. (3 marks)

- 7 The diagram shows the graph of $y = \cos 2x$ for $0^\circ \leq x \leq 360^\circ$.

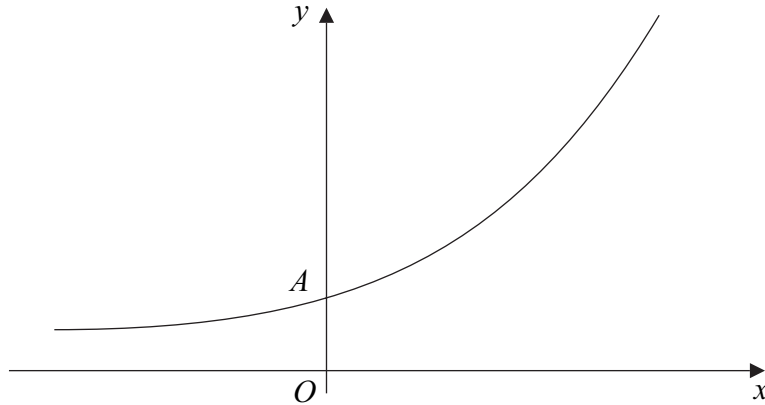


- (a) Write down the coordinates of the points A , B and C marked on the diagram. (4 marks)
- (b) Describe the single geometrical transformation by which the curve with equation $y = \cos 2x$ can be obtained from the curve with equation $y = \cos x$. (2 marks)
- (c) Solve the equation

$$\cos 2x = 0.37$$

giving all solutions to the nearest 0.1° in the interval $0^\circ \leq x \leq 360^\circ$. (No credit will be given for simply reading values from a graph.) (5 marks)

- 8 The diagram shows a sketch of the curve with equation $y = 3^x + 1$.

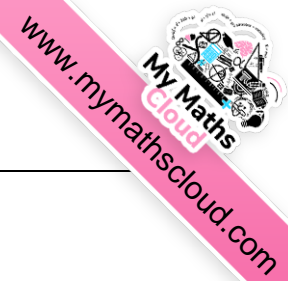


The curve intersects the y -axis at the point A .

- (a) Write down the y -coordinate of point A . (1 mark)
- (b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximation for $\int_0^1 (3^x + 1) dx$, giving your answer to three significant figures. (4 marks)
- (ii) By considering the graph of $y = 3^x + 1$, explain with the aid of a diagram whether your approximation will be an overestimate or an underestimate of the true value of $\int_0^1 (3^x + 1) dx$. (2 marks)
- (c) The line $y = 5$ intersects the curve $y = 3^x + 1$ at the point P . By solving a suitable equation, find the x -coordinate of the point P . Give your answer to four decimal places. (4 marks)
- (d) The curve $y = 3^x + 1$ is reflected in the y -axis to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. (1 mark)

END OF QUESTIONS

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